BOOK REVIEWS

"The scope of this book is modest: we study no Lie algebras with dimension greater than 6. Furthermore, in the six-dimensional case,  $\mathfrak{I}_6$ , we do not give complete results. (The so-called addition theorems of Gegenbauer type for Bessel functions would be obtained from such an analysis.) However, it should be clear to the reader that our methods can be generalized to higher dimensional Lie algebras.

"We will almost exclusively be concerned with the derivation of recursion relations and addition theorems. The manifold applications of group theory in the derivation of orthogonality relations and integral transforms of special functions will rarely be considered. For these applications see the encyclopedic work of Vilenkin. The overlap in subject matter between that book and this one is relatively small, except in the study of unitary representations of real Lie groups."

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**26[7].**—RUDOLF ONDREJKA, *Exact Values of*  $2^n$ , n = 1(1)4000, ms. of 519 computer sheets deposited in the UMT file.

This impressive table of the exact values of the first 4000 powers of 2, which was computed in 1961 on an IBM 709 system, constitutes the first volume of an extensive unpublished series of such tables.

In private correspondence with the editors the author has revealed that in October and November 1966 and in the period from May through August 1967 he extended this initial table by computing the next 29,219 powers of 2 on an IBM 7090 system. These additional powers occupy a total of 27,023 computer sheets, arranged in 74 volumes, which are in the possession of the author.

A further statistic supplied by the author is that the total number of digits in all 75 volumes is 166,115,268. This digit count is also given for each volume; thus, the volume under review, for example, contains 2,410,843 digits. A useful index has been supplied for the entire set of tables.

The selection of 33,219 as the total number of entries in this immense tabulation was based on the author's plan to include all powers of 2 whose individual lengths do not exceed 10,000 decimal digits.

The tabular entries are clearly printed in groups of five digits, with 19 such pentads on each line. The appropriate exponent is printed beside each entry, and successive powers are conveniently separated by a double space.

In a brief abstract the author has added the information that tabular values were spot checked by comparison with the published results of others, particularly those relating to the known Mersenne primes.

J. W. W.

27[7].—H. R. AGGARWAL & VAHE SAGHERIAN, An Extension of the Tables of the Quotient Functions of the Third Kind, Stanford Research Institute, Menlo Park, Calif., ms. of 18 typewritten pages deposited in the UMT file. The quotient functions of the third kind refer herein to the ratios

$$h_{\nu}^{(1)}(z) = z H_{\nu-1}^{(1)}(z) / H_{\nu}^{(1)}(z)$$
 and  $h_{\nu}^{(2)}(z) = z H_{\nu-1}^{(2)}(z) / H_{\nu}^{(2)}(z)$ ,

where  $H_{\nu}^{(1)}(z)$  and  $H_{\nu}^{(2)}(z)$  are the Hankel functions of the first and second kinds, respectively, of order  $\nu$ .

The tables consist of 5D values of  $[2/(2n - 1)] \Re[h_n^{(1)}(x)]$  and  $\mathfrak{g}[h_n^{(1)}(x)]$  for n = 0(1)10, x = 0(0.2)15. Graphs of the tabulated quantities are included. The ratio  $h_r^{(2)}(x)$  is not tabulated, since it is merely the complex conjugate of  $h_r^{(1)}(x)$ .

Appropriate reference is made to the related tables of Onoe [1], [2].

The authors include a discussion of the asymptotic forms of the functions  $h_{\nu}^{(1)}(z)$  and  $h_{\nu}^{(2)}(z)$ , and they state that such functions arise in the study of the propagation of sinusoidal sound waves in loud-speaker horns and in the diffraction of steady plane elastic waves by circular cylindrical discontinuties embedded in an infinite medium.

## J. W. W.

2. M. ONOE, Tables of Modified Quotients of Bessel Functions of the First Kind for Real and Imaginary Arguments, Columbia Univ. Press, New York, 1958. (See MTAC, v. 13, 1959, p. 131, RMT 22.)

## 28[7, 13.05].—SHIGETOSHI KATSURA & KATSUHIRO NISHIHARA, Tables of Integrals of Products of Bessel Functions. II, Department of Applied Physics, Tôhoku University, Sendai, Japan, undated, ms. of 21 computer sheets deposited in the UMT file.

These manuscript tables, which constitute a sequel to earlier manuscript tables by Kilpatrick, Katsura & Inoue [1], gives 16S decimal values (in floating-point form) to the coefficients of  $p^{l}$  in the polynomial expression for the integral

$$\int_{0}^{\infty} J_{1/2}(pt) J_{3/2+m}(bt) J_{3/2+n}(ct) t^{-3/2} dt$$

for  $l = -\frac{1}{2}$  (1) m + n + 7/2; m, n = 0(1)6, m + n even; b, c = 1 and 2. These data were calculated on an IBM 7090, using double-precision arithmetic. A spot check revealed that several entries are accurate to only 13S.

Details of the evaluation of the integral by the calculus of residues are set forth in the introductory text of seven pages, to which is appended a list of eight references.

These tables have been used by the authors in the calculation of the molecular distribution function for a square-well potential gas.

J. W. W.

M. ONOE, "Formulae and tables, the modified quotients of cylinder functions," Report of the Institute of Industrial Science, University of Tokyo, v. 4, 1955, pp. 1-22. (See MTAC, v. 10, 1956, p. 53, RMT 29.)
M. ONOE, Tables of Modified Quotients of Bessel Functions of the First Kind for Real and

<sup>1.</sup> J. E. KILPATRICK, SHIGETOSHI KATSURA & YUJI INOUE, Tables of Integrals of Products of Bessel Functions, Rice University, Houston, Texas and Tôhoku University, Sendai, Japan, 1966. (See Math. Comp., v. 21, 1967, p. 267, UMT 27.)